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Numerical study on equilibrium stability of objects in fluid flow — A case study on constructal law

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ABSTRACT

Stability of objects in fluid flow is an interesting and significant subject in many fields. From thermodynamics point of view, the constructal law by Bejan et al. proposed that in some scenario, objects tend to be stable when the drag reaches maximum. To investigate the relation between drag and stable positions, we analyzed two simple cases by the finite element method: an elliptical cylinder and a rectangular cylinder immersed in 2D uniform laminar flow. The ellipse is stable when its major axis is perpendicular to the mainstream and the drag reaches maximum; yet the rectangle is stable when the length is perpendicular to the mainstream, with drag ranging from minimum to maximum depending on its aspect ratio. Our results show that there is no universal relation between drag and stability.

1. Introduction

The research on the stability of objects in fluid flow has great academic and industrial significance in many fields. The motion of fluids satisfies the Navier-Stokes equation and shows nonlinear characteristics. Thus, stability analysis of objects in fluid flow is very complicated.

Constructal law, proposed by Bejan et al., is claimed to be a universal thermodynamic law which states: "for a finite-size flow system to persist in time to survive, its configuration must evolve in such a way that it provides an easier access to the currents that flow through it." [1] Constructal law has been used in many fields, especially the design of heat exchangers [2,3]. According to the constructal law, they proposed a criterion for stable positions of floating objects, stating that an object in flow field is stable when the drag reaches maximum [4]. As shown in Fig. 1, an object (iceberg or tree log) floats on the surface of the ocean. The atmosphere (a) moves with the wind speed U_a, while the ocean water (b) is stationary. If we consider (a)+(b) to be a system, the constructal law requires the object to bring (a) and (b) to equilibrium the fastest, namely the object stabilizes where it reaches maximum drag. The stable position of the cylinder is (1) only, although both (1) and (2) positions are equilibrium.

In this paper, we try to find the relation between drag and stable positions. We analyzed the stable positions of an ellipse and a rectangle in uniform laminar flow by the finite element method. Our results show that stable positions of objects do not necessarily correspond to maximum drag.

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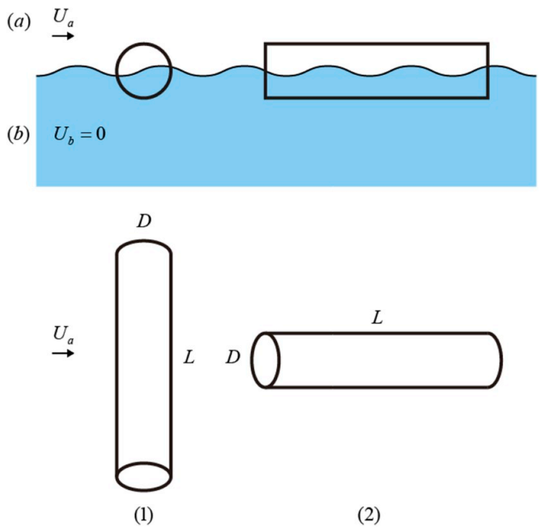

Fig. 1. Floating object at the interface between two fluid masses with relative motion (redrawn from Fig. 8 of Ref. [4]).

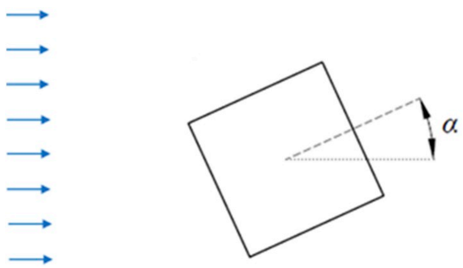

Fig. 2. An object with fixed centroid in uniform flow.

2. Methods

2.1. Criterion of stable positions

In this paper, we focus on systems with single degree of freedom, specifically, objects with fixed centroids and rotational freedom. As shown in Fig. 2, the object is only able to rotate about its fixed centroid. Our goal for the stable position is equivalent to looking for the stable angle α.

Considering simple configuration and relatively fast response, we deduced a criterion to determine the object's stable position based on quasi-steady assumptions [5].

Generally, stability depends on forces applied to the objects. For our system, stability depends on moment *M*, assumed to be a function of angle of attack α and angular velocity ω:

$$M = M(\alpha, \omega) \tag{1}$$

Let $\alpha = \alpha'$ be an equilibrium position of the object, then the moment about its fixed centroid is zero, namely, $M(\alpha',0) = 0$. We need to determine whether it is stable. When the object is disturbed, it has angular displacement $\Delta\alpha$ and angular velocity $\Delta\omega$, and moves away from the equilibrium position ($\Delta\alpha\Delta\omega > 0$). Due to small disturbance, we have

$$M\left(\alpha' + \Delta\alpha, \Delta\omega\right) = \frac{\partial M(\alpha,\omega)}{\partial \alpha}\bigg|_{(\alpha',0)} \Delta\alpha + \frac{\partial M(\alpha,\omega)}{\partial w}\bigg|_{(\alpha',0)} \Delta\omega \tag{2}$$

Dividing Eq. (2) by $\Delta\alpha$:

$$\frac{M(\alpha' + \Delta\alpha, \Delta\omega)}{\Delta\alpha} = \frac{\partial M(\alpha,\omega)}{\partial \alpha}\bigg|_{(\alpha',0)} + \frac{\partial M(\alpha,\omega)}{\partial w}\bigg|_{(\alpha',0)} \frac{\Delta\omega}{\Delta\alpha} \tag{3}$$

We define that when the moment is in the opposite direction to disturbance, the object is regarded as stable; otherwise it is unstable. This definition accords with the concept of "static stability" used in aircrafts [6]. Therefore, to be stable we require

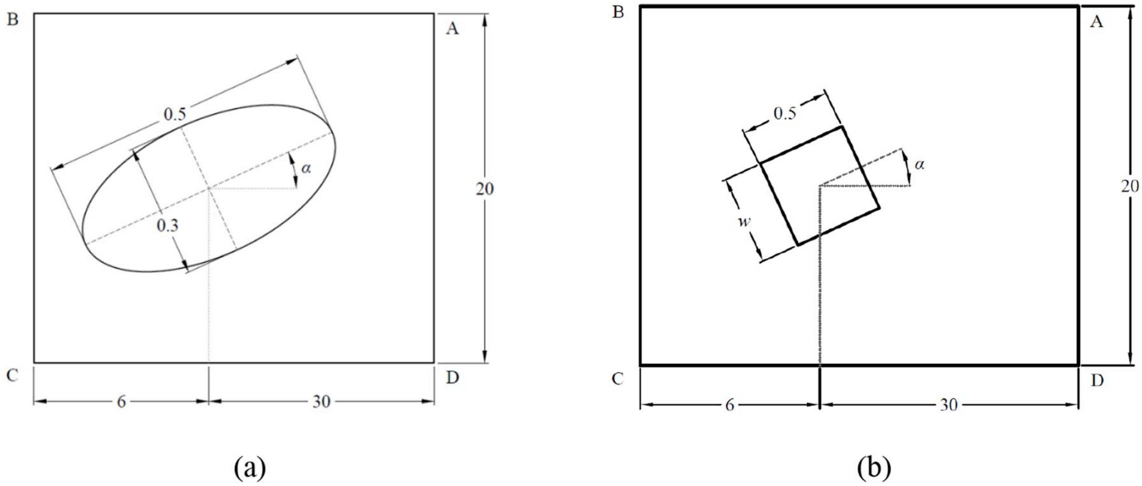

Fig. 3. Calculating domain for (a) an elliptical cylinder, (b) a rectangular cylinder (not to scale, unit: m).

$$\forall \frac{\Delta\omega}{\Delta\alpha} > 0, \frac{M(\alpha' + \Delta\alpha, \Delta\omega)}{\Delta\alpha} < 0 \tag{4}$$

It is assumed that angular velocity always induces an opposite moment:

$$\left.\frac{\partial M(\alpha,\omega)}{\partial w}\right|_{(\alpha',0)} < 0 \tag{5}$$

Combining Eqs. (3)–(5), the equivalent condition for stableness is that angular displacement caused by disturbance always induces an opposite moment:

$$\left.\frac{\partial M(\alpha,\omega)}{\partial \alpha}\right|_{(\alpha',0)} < 0 \tag{6}$$

Eq. (6) simplifies complicated transient analysis as a steady problem. The moment is only related to angle α, so we just need to calculate the steady fluid field after the object rotates by a small angle about its centroid. An equilibrium position is stable if the moment is opposite to the small angle by which the object rotates, and vice versa.

2.2. Numerical simulation

We tried two simple shapes to investigate the relation between drag and stability: an ellipse cylinder and a rectangular cylinder in 2D uniform flow. The finite element method was used to calculate the fluid fields, implemented by COMSOL Multiphysics software. For each geometry, we calculated the steady fluid field varying the angle of attack α, and recorded the corresponding drag, namely force applied to the object along mainstream, and moment about the object's centroid.

The calculating domains are shown in Fig. 3. In either subfigure, segment BC is the *inlet*; the ellipse or rectangle denotes the *wall*; segment AD is the *pressure outlet*.

It is worthy to note that to avoid possible singularity caused by sharp angles of the rectangle, each corner has a fillet whose radius is 1/20 of the width, which is not shown in the figure.

3. Results

3.1. Elliptical cylinder

We defined the Reynolds number with major axis length $2a$ as the characteristic length:

$$\mathrm{Re} = \frac{2\rho a U}{\mu} \tag{7}$$

where ρ denotes density, μ denotes viscosity, a denotes semi-major axis length and U denotes mainstream speed.

Drag coefficient was defined by

$$C_D = \frac{F}{\frac{1}{2}2a\rho U^2} = \frac{F}{a\rho U^2} \tag{8}$$

Table 1
Parameters for uniform flow past an elliptical cylinder.

No	Semi major axis a/m	Semi minor axis b/m	Velocity $U/(\mathrm{m{\cdot}s^{-1}})$	Density $\rho/(\mathrm{kg{\cdot}m^{-3}})$	Viscosity $\mu/(\mathrm{N{\cdot}s{\cdot}m^{-2}})$	Reynolds number Re
1	0.5	0.3	1	10	1	10
2	0.5	0.3	1	50	1	50

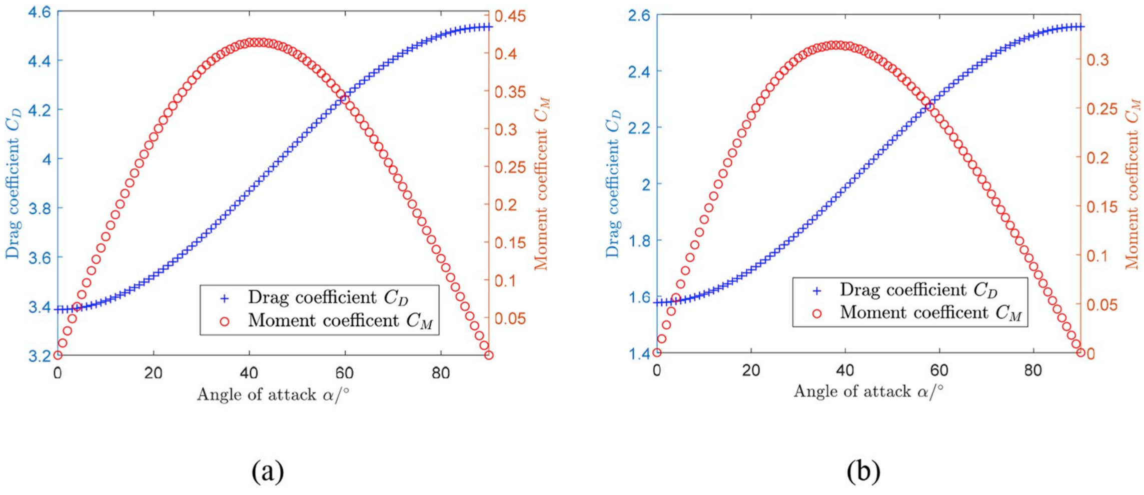

Fig. 4. Drag and moment vs. angle of attack for the elliptical cylinder when (a) Re = 10, (b) Re = 50.

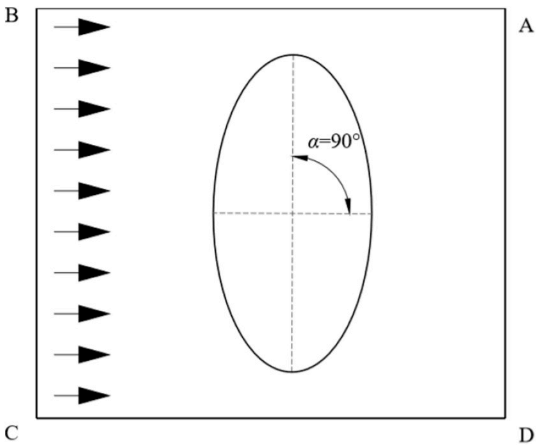

Fig. 5. The stable position of the elliptical cylinder.

where F denotes drag force.

Similarly, the moment coefficient was defined as

$$C_M = \frac{M}{\frac{1}{2}\rho(2a)^2U^2} = \frac{M}{2\rho a^2U^2} \tag{9}$$

where M denotes moment.

Parameters used in the simulation are presented in Table 1; the physical properties were designed to achieve required Reynolds numbers. We aimed to simulate steady flow, so we only tried configurations with low Reynolds numbers to avoid large Karman vortices and even turbulent flow.

As shown in Fig. 4, the trends of drag coefficient C_D and moment coefficient C_M follow the same pattern when angle α changes despite different Reynolds numbers. When α increases, the actual frontal area of the elliptical cylinder increases so the drag increases as well. The moment remains positive (counterclockwise) except for $\alpha = 0°$ or $\alpha = 90°$, which means the moment rotates the cylinder

Table 2
Parameters for uniform flow past a rectangular cylinder.

No	Length l/m	Width w/m	Velocity $U/(\mathrm{m{\cdot}s^{-1}})$	Density $\rho/(\mathrm{kg{\cdot}m^{-3}})$	Viscosity $\mu/(\mathrm{N{\cdot}s{\cdot}m^{-2}})$	Reynolds number Re
1	0.5	0.05,0.25,0.4,0.5	1	20	1	10
2	0.5	0.05,0.25,0.4,0.5	1	100	1	50

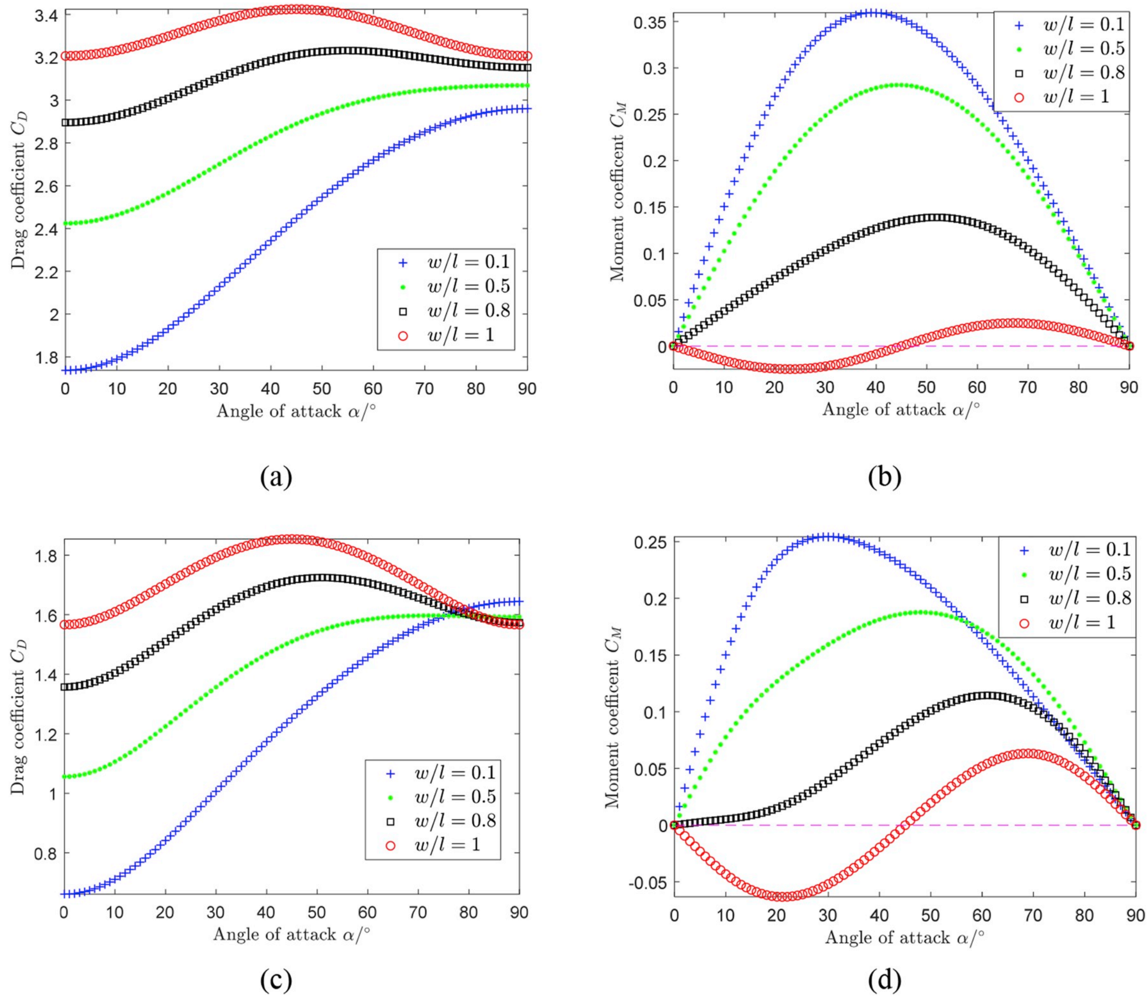

Fig. 6. Coefficients vs. angle of attack for the rectangular cylinder of (a) drag, Re = 10, (b) moment, Re = 10, (c) drag, Re = 50, (d) moment, Re = 50.

towards the position where $\alpha = 90^{\circ}$. Therefore, despite two equilibrium positions ($\alpha = 0^{\circ}$ and $\alpha = 90^{\circ}$), the cylinder is only stable when $\alpha = 90^{\circ}$, since any disturbance at $\alpha = 0^{\circ}$ will induce a moment rotating the cylinder to the other equilibrium position. At the stable position, the major axis is perpendicular to the mainstream and the drag reaches maximum, shown in Fig. 5.

3.2. Rectangular cylinder

Like the elliptical cylinder, we defined the Reynolds number

$$\mathrm{Re} = \frac{\rho l U}{\mu} \tag{10}$$

where l denotes the length of the rectangle.

Drag coefficient C_D and moment coefficient C_M were defined by Eq. (11) and Eq. (12):

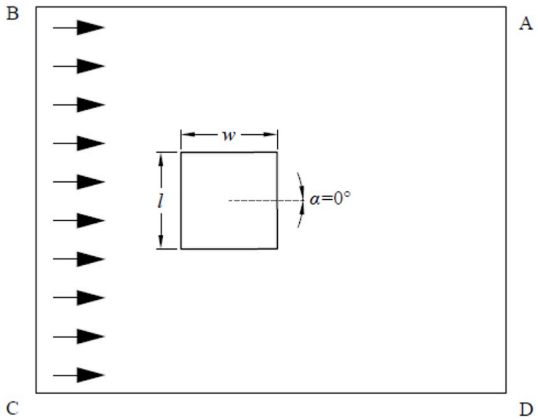

Fig. 7. The stable position of the rectangular cylinder ($w \leq l$).

$$C_D = \frac{F}{\frac{1}{2} l \rho U^2} \tag{11}$$

$$C_M = \frac{M}{\frac{1}{2} \rho l^2 U^2} \tag{12}$$

Parameter setting is shown in Table 2; again, the physical properties were designed intentionally. We tested the cases with four various aspect ratios and two Reynolds numbers.

It can be observed from Fig. 6 that, similar to the elliptical cylinder, Reynolds number does not impact the relation between drag and moment for a given aspect ratio. In Fig. 6(b) and (d), the moment is always positive when $w/l < 1$, rotating the object counterclockwise until $\alpha = 90^\circ$; when $w/l = 1$, the rectangle becomes a square, the moment is negative (clockwise) when $\alpha < 45^\circ$, and rotates the square towards $\alpha = 0^\circ$ and $\alpha = 90^\circ$. In another word, the stable positions of a rectangular cylinder are always the same despite different aspect ratios: length must be perpendicular to the mainstream (Fig. 7); a square cylinder, as a special case where the length is equal to the width, can be stable when either edge is perpendicular to the mainstream, but is not stable when the diagonal is perpendicular to the mainstream ($\alpha = 45^\circ$) despite an equilibrium position there. Drag is more sensitive to aspect ratios, as shown in Fig. 6(a) and (c). When $w/l < 0.5$, drag reaches maximum at $\alpha = 90^\circ$. As width w grows, the angle with maximum drag decreases until $\alpha = 45^\circ$ when $w/l = 1$. That means, the stable positions of a rectangular cylinder can correspond to maximum drag, minimum drag, or somewhere between them.

4. Conclusions

Inspired by a corollary of the construtal law, we tried to investigate the relation between drag and stable positions of objects in fluid flow. Our focus is on two simple examples: an elliptical cylinder and a rectangular cylinder with fixed centroids in uniform mainstream. Based on quasi-steady assumptions and the concept of "static stability", we derived a stability criterion: stability is determined by the direction of resultant moment at steady state when the object rotates by a small angle about its centroid.

Both the elliptical cylinder and rectangular cylinder have two equilibrium positions when rotating from one symmetry axis to the next, yet only one stable position. The elliptical cylinder is stable when its major axis is perpendicular to the mainstream and the drag reaches maximum. However, the rectangular cylinder is stable when its length is perpendicular to the mainstream with drag depending on the aspect ratio: the drag is maximum for a slim rectangle ($w/l < 0.5$), minimum for a square ($w/l = 1$) and in the middle for a broad rectangle ($0.5 < w/l < 1$).

Our results reveal that there is no universal relation between drag and stability. For a slim object, such as the circular cylinder in Fig. 1, an ellipse and a slim rectangle, stable positions tend to correspond to maximum drag. In contrast, for some geometries, such as a square, stable positions may correspond to minimum drag; for others, stable positions may also correspond to drag in the middle.

Declaration of competing interest

The authors declared that there is no conflict of interest.

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